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Higher-spin Realisations of the Bosonic String [1]

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Abstract

It has been shown that certain W algebras can be linearised by the inclusion of a spin–1 current. This provides a way of obtaining new realisations of the W algebras. Recently such new realisations of W_3 were used in order to embed the bosonic string in the critical and non-critical W_3 strings. In this paper, we consider similar embeddings in $W_{2,4}$ and $W_{2,6}$ strings. The linearisation of $W_{2,4}$ is already known, and can be achieved for all values of central charge. We use this to embed the bosonic string in critical and non-critical $W_{2,4}$ strings. We then derive the linearisation of $W_{2,6}$ using a spin–1 current, which turns out to be possible only at central charge $c = 390$. We use this to embed the bosonic string in a non-critical $W_{2,6}$ string.

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With the discovery of the property of the W_3 algebra that it can be linearised by the inclusion of a spin–1 current [1], new realisations were constructed for the purpose of building the corresponding W_3 strings [2, 3]. An unusual feature of these realisations is that the spin–3 current contains a term linear in a ghost-like field. The realisations also close when this term is omitted, under which circumstance the corresponding string theory is equivalent to the one that is based on the Romans' free-scalar realisation [4]. However, when this term is included, the corresponding BRST operator is equivalent to that of the bosonic string, which can be shown by making a local canonical field redefinition [2]. Thus the new realisations provide embeddings of the bosonic string in the W_3 string.

It is interesting to generalise the above consideration to the embedding of the bosonic string in $W_{2,s}$ strings, where $W_{2,s}$ denotes the conformal algebra generated by a spin–s current together with the energy-momentum tensor. The $W_{2,s}$ strings based on free-scalar realisations were extensively discussed in Ref. [5], where it was shown that when $s \geq 3$ the cohomologies describe Virasoro strings coupled to certain minimal models. The $W_{2,s}$ algebras exist at the classical level for all positive integer values of s. However, at the quantum level, for generic values of s, a $W_{2,s}$ algebra exists only for a finite set of special values of central charge [6, 7], which in particular does not include the critical value. The exceptions are the $W_{2,s}$ algebras for $s = 1, 2, 3, 4$ and 6, for which the central charge can be arbitrary. Although the $W_{2,s}$ algebra does not close at the critical central charge for generic values of s, it is nevertheless possible to build $W_{2,s}$ strings with free-scalar realisations. It was shown in Ref. [8] that one can first use the free-scalar realisation to write down the classical BRST operator, and then quantise the theory by renormalising the transformation rules and adding necessary quantum counter-terms.

The new realisations that were constructed in Ref. [2], which provide embeddings of the bosonic string in $W_3 = W_{2,3}$ strings, do not generate the W_3 algebra at the classical level. The W_3 symmetry arises only as a consequence of quantisation. Thus it seems that if we are to use such new realisations for values of s other than 3, we must restrict our attention to the cases $s = 1, 2, 4$ and 6, for which the quantum algebras exist. The embeddings of the bosonic string in the $W_{2,s}$ string for $s = 1, 2$ and 3 were discussed in Ref. [2]. In this paper, we shall focus our attention on the remaining cases $s = 4$ and 6.

It is instructive to begin by studying the form of the linearisation of the W_3 and $W_{2,4}$ algebras, for which the results were obtained in Refs. [1, 9]. The associated linearised $W_{1,2,3}$

and $W_{1,2,4}$ algebras take the form:

$$
\begin{aligned}
T_0(z)\,T_0(0) &\sim \frac{c}{2z^4}+\frac{2T_0}{z^2}+\frac{\partial T_0}{z}\ , \qquad T_0(z)\,W_0(0)\sim\frac{sW_0}{z^2}+\frac{\partial W_0}{z}\ ,\\
T_0(z)\,J_0(0) &\sim \frac{c_1}{z^3}+\frac{J_0}{z^2}+\frac{\partial J_0}{z}\ , \qquad J_0(z)\,J_0(w)\sim-\frac{1}{z^2}\ ,\\
J_0(z)\,W_0(w) &\sim \frac{hW_0}{z}\ , \qquad W_0(z)\,W_0(w)\sim 0\ ,
\end{aligned}
\tag{1}
$$

where $s=3$ and 4 respectively. The coefficients c, c_1 and h are given by

$$
\begin{aligned}
&c=50+24t^2+\frac{24}{t^2}\ , \qquad c_1=-\sqrt{6}(t+\frac{1}{t})\ , \qquad h=\sqrt{\tfrac{3}{2}}t\ , \qquad (s=3)\\
&c=86+30t^2+\frac{60}{t^2}\ , \qquad c_1=-3t-\frac{4}{t}\ , \qquad h=t\ . \qquad (s=4)
\end{aligned}
\tag{2}
$$

The currents of the W_3 and $W_{2,4}$ algebras are then given by

$$
T=T_0\ , \qquad W=W_0+W_{\rm R}\ , \tag{3}
$$

where $W_{\rm R}$ is the Romans type realisation constructed from T_0 and J_0. For the cases where $s=3,4$ and 6, $W_{\rm R}$ takes the form [8]

$$
W_{\rm R}=\sum_{n=0}^{[s/2]} g_n(s)J_0^{s-2n}\,T_0^n+\hbox{quantum corrections}\ , \tag{4}
$$

where the g_n's are given by

$$
g_n=\frac{(-2)^{s/2}(s-n-1)!}{2^n n!(s-2n)!}\ . \tag{5}
$$

The coefficients c and c_1 can be determined from the background charges of the free-scalar realisation. In terms of the two scalar realisation, the energy-momentum tensor can be expressed as

$$
T_0=-\tfrac{1}{2}(\partial\vec\phi)^2-(t\vec\rho+\tfrac{1}{t}\vec\rho^{\vee})\cdot\partial^2\vec\phi\ , \tag{6}
$$

where $\vec\phi=(\phi_1,\phi_2)$. The vectors $\vec\rho$ and $\vec\rho^{\vee}$ are the Weyl vector and co-Weyl vector for the Lie algebras A_2 and B_2, associated with W_3 and $W_{2,4}$ respectively. For the A_2 algebra, we have $\vec\rho=\vec\rho^{\vee}=(\sqrt{\tfrac{3}{2}},\sqrt{\tfrac{1}{2}})$; for B_2, $\vec\rho=(\tfrac{3}{2},\tfrac{1}{2})$ and $\vec\rho^{\vee}=(2,1)$. In each case, the scalar ϕ_2 occurs in the higher-spin current only *via* the energy-momentum tensor T_0, and hence its contribution can be replaced by an arbitrary effective energy-momentum tensor. The scalar ϕ_1 thus plays a distinguished rôle. It has a background charge $\alpha=-\sqrt{\tfrac{3}{2}}(t+t^{-1})$ for W_3 and $\alpha=-\tfrac{3}{2}t-2t^{-1}$ for $W_{2,4}$. Thus $c_1=2\alpha$ in each case. The central charge c follows immediately from Eqn. (6). That $c_1=2\alpha$ is not coincidental. In fact one can realise the algebra given in Eqn. (1) by the

two scalar realisation, with $J_0 = \partial\phi_1$, $W_0 = 0$ and T_0 given by Eqn. (6). The third-order pole in the OPE $T_0(z)J_0(0)$ is then precisely 2α.

We now turn to the case of $W_{2,6}$. With the coefficients c, c_1 and h undetermined, the form given by Eqn. (1) with $s = 6$ is the most general possible for the linearised $W_{1,2,6}$ algebra. *A priori*, one might have expected, without loss of generality, that there could be linear terms involving the currents J_0, T_0 and their derivatives in the OPE $J_0(z)W_0(0)$. However, we have verified that these terms are excluded by the requirement of closure of the $W_{1,2,6}$ algebra. To determine the coefficients c, c_1 and h, we use the realisation (3) to construct the $W_{2,6}$ quantum algebra. The most general form for $W_{\rm R}$ in this case has 29 terms. The requirement that W in Eqn. (3) be primary determines all but three of the associated coefficients. The remaining coefficients can be determined by studying the OPE $(W_0(z)W_{\rm R}(0) + W_{\rm R}(z)W_0(0))$, in which all terms involving J_0 have to be zero. This determines all the rest of the coefficients, including c, c_1 and h. Unlike the previous cases of $W_{2,3}$ and $W_{2,4}$, where the coefficients c, c_1 and h are expressed in terms of the free parameter t in Eqn. (2), here these coefficients are uniquely determined, modulo a trivial reflection symmetry $J_0 \longrightarrow -J_0$, namely $c = 390, c_1 = 11$ and $h = -1$. One might have expected, since the Weyl vector and co-Weyl vector of the Lie algebra G_2 associated with the $W_{2,6}$ algebra are $\vec\rho = (\frac{3}{2}, \sqrt{\frac{1}{12}})$ and $\vec\rho^\vee = (5, \sqrt{3})$, that one could express the central charges as $c = 194 + 28t^2 + 336t^{-2}$ and $c_1 = -3t - 10t^{-1}$. However the solution we have found implies that this is true only at $t = -2$, corresponding to the central charge $c = 390$. The spin–6 current W is given by

$$
\begin{aligned}
W \;=\;& W_0 - \tfrac{1}{6}J_0^6 - \tfrac{1}{2}T_0J_0^4 - \tfrac{4921}{114718}T_0^3 - \tfrac{3}{8}T_0^2J_0^2 + \tfrac{9}{8}T_0^2\partial J_0 + \tfrac{15}{2}T_0\partial J_0\, J_0^2 - \tfrac{21}{2}T_0(\partial J_0)^2 \\
& -\tfrac{41}{4}T_0\partial^2 J_0\, J_0 + \tfrac{21}{4}T_0\partial^3 J_0 + \tfrac{11}{2}\partial J_0\, J_0^4 - \tfrac{315}{8}(\partial J_0)^2 J_0^2 + \tfrac{277}{8}(\partial J_0)^3 + \tfrac{7}{4}\partial T_0\, J_0^3 \\
& +\tfrac{3}{2}\partial T_0\, T_0 J_0 - \tfrac{57}{4}\partial T_0\, \partial J_0\, J_0 - \tfrac{190257}{229436}(\partial T_0)^2 + \tfrac{43}{4}\partial T_0\, \partial^2 J_0 - \tfrac{157}{12}\partial^2 J_0\, J_0^3 \\
& +\tfrac{409}{4}\partial^2 J_0\, \partial J_0\, J_0 - \tfrac{1763}{48}(\partial^2 J_0)^2 - \tfrac{108753}{114718}\partial^2 T_0\, T_0 - \tfrac{45}{16}\partial^2 T_0\, J_0^2 + \tfrac{135}{16}\partial^2 T_0\, \partial J_0 \\
& +\tfrac{273}{16}\partial^3 J_0\, J_0^2 - \tfrac{787}{16}\partial^3 J_0\, \partial J_0 + \tfrac{5}{2}\partial^3 T_0\, J_0 - \tfrac{197}{16}\partial^4 J_0\, J_0 - \tfrac{440915}{458872}\partial^4 T_0 + \tfrac{383}{96}\partial^5 J_0\ .
\end{aligned}
\tag{7}
$$

It may be that a linearisation of $W_{2,6}$ for arbitrary central charge is possible if further currents are added.

Now let us turn our attention to the study of the $W_{2,s}$ strings. Eqn. (3) provides new realisations of the $W_{2,s}$ algebras, for $s = 1, 2, 3, 4$ and 6. If the current W_0 is zero, then the resulting realisation is precisely the same as the free-scalar realisation, with the distinguished scalar $\partial\phi_1$ replaced by the abstract spin–1 current J_0. However it was shown, for the cases of $s = 3, 4$, that the current W_0 does not have to be zero, and that instead it could be realised by

a parafermionic vertex operator [1, 9]. One can alternatively realise W_0 in terms of a ghost-like field [2, 3]. It was shown in Ref. [2], by performing local canonical field redefinitions, that for the latter realisations, with $s = 1, 2, 3$, the corresponding $W_{2,s}$ strings are equivalent to the bosonic string. In this letter, we shall construct new realisations involving ghost-like fields for $W_{2,4}$ and $W_{2,6}$, and argue that these realisations provide embeddings of the bosonic string in the corresponding W strings.

First let us consider the $W_{2,4}$ case. To obtain a realisation for the linearised $W_{1,2,4}$ algebra (1), we introduce a pair of bosonic ghost-like fields (r, s) with spins $(4, -3)$, and a pair of fermionic ghost-like fields (b_1, c_1) with spins $(k, 1 - k)$. The realisation is then given by

$$\begin{aligned} T_0 &= T_X + 4r\partial s + 3\partial r\, s - k\, b_1\partial c_1 - (k-1)\partial b_1\, c_1 \ , \\ W_0 &= r \ , \qquad J_0 = -t\, rs + \sqrt{t^2-1}\, b_1 c_1 \ , \end{aligned} \tag{8}$$

where $(2k - 1)^2 = 16(1 - t^{-2})$, and T_X is an arbitrary energy-momentum tensor with central charge $c_X = -13+30t^2+12t^{-2}$. The total central charge of the realisation is $c = 86+30t^2+60t^{-2}$. Once having obtained a realisation of $W_{1,2,4}$, one can obtain a realisation for $W_{2,4}$ of the form (3) by a basis change [9]. Note that when $t^2 = 1$, the (b_1, c_1) term is absent from J_0, and thus it can be absorbed into T_X, giving rise to effective central charge $c_X = 30$. In this case, the realisation takes its simplest form. The corresponding total central charge is $c = 176$.

To quantise a $c = 176$ $W_{2,4}$ string, we need to use a non-critical BRST construction, in which we introduce a $c = -4$ Liouville sector, since the critical central charge for the $W_{2,4}$ string is $c = 172$. The non-critical BRST operator for $W_3 = W_{2,3}$ was first obtained in Ref. [10]. Subsequently, the non-critical $W_{2,4}$ BRST operator was constructed in Ref. [11]. The two-scalar realisation of the $W_{2,4}$ algebra was first constructed in Ref. [6]. We can instead realise the $W_{2,4}$ algebra by two pairs of fermionic fields (b_2, c_2) and (b_3, c_3). When $c = -4$, the realisation takes the particularly simple form

$$\begin{aligned} T_{\rm L} &= -b_2\partial c_2 - b_3\partial c_3 \ , \\ W_{\rm L} &= b_2\partial c_2\, b_3\partial c_3 + \tfrac{5}{2}(T_{\rm L}^2 - \tfrac{3}{10}\partial^2 T_{\rm L}) \ . \end{aligned} \tag{9}$$

Note that this realisation does not close classically, but it does close at the quantum level. If one bosonises the (b_2, c_2) and (b_3, c_3) fields, it is equivalent to the two-scalar realisation. With this realisation for the $c = -4$ Liouville sector, one can write down the non-critical $W_{2,4}$ BRST operator in the graded form [11]

$$Q_0 \;=\; \oint c\Big(T + T_{\rm L} - 4\,\beta\partial\gamma - 3\partial\beta\,\gamma - b\partial c\Big) \ ,$$

$$
\begin{aligned}
Q_1 &= \oint \gamma\Big(195\sqrt{\tfrac{2}{451}}W - \tfrac{59}{451}T^2 + b_2\partial c_2\, T - 4b_2\partial c_2\,\beta\partial\gamma + T\beta\partial\gamma - \tfrac{298}{451}\partial^2 T \\
&\quad +3b_2\partial^3 c_2 + 5\partial b_2\,\partial^2 c_2 + 3\partial^2 b_2\,\partial c_2 + 3\beta\partial^3\gamma + 2\partial\beta\,\partial^2\gamma\Big) \ ,
\end{aligned} \tag{10}
$$

where (c, b) and (γ, β) are the ghost fields for the spin–2 and spin–4 currents respectively, and T and W generate the $W_{2,4}$ algebra with $c = 176$. It is interesting that this non-critical BRST operator with abstract matter currents has a simpler form than the abstract critical $W_{2,4}$ BRST operator [12]. Substituting the $c = 176$ realisation that we discussed above, the Q_0 operator has the same form, with $T = T_X + 4r\partial s + 3\partial r\, s$, and the Q_1 operator can be expressed as

$$
Q_1 = \oint \gamma\Big(r - \tfrac{1}{4}r^4 s^4 + \text{more}\Big) \ , \tag{11}
$$

modulo an overall constant factor, where the "more" terms are quantum corrections to the classical terms $\gamma(r - \frac{1}{4}r^4 s^4)$. Note that the Liouville sector enters the Q_1 operator only as a quantum correction. The Q_1 operator is analogous to the one for the W_3 string, where $Q_1 = \oint \gamma(r - \frac{1}{3}r^3 s^3 + \text{quantum corrections})$ [2]. It was shown that the Q_1 operator for W_3 can be converted into a single term γr by a local canonical field redefinition. We expect that this can also be done for the Q_1 operator (11) for $W_{2,4}$. To see this, we note that the classical terms $\gamma(r - \frac{1}{j}r^j s^j)$ can be converted into the single term γr by the following local field redefinition

$$
\begin{aligned}
r &\longleftarrow r - \tfrac{1}{j}r^j s^j \ , \\
s &\longleftarrow \sum_{n\ge 0} g_n r^{nj-n} s^{nj+1} \ ,
\end{aligned} \tag{12}
$$

where $g_n = n(nj+1)^{-1} g_{n-1}$ with $g_0 = 1$. We expect that the operator Q_1 in Eqn. (11) can also be converted into the single term by local field redefinitions at the full quantum level. Since the redefined (r, s) and (β, γ) fields then form a Kugo-Ojima quartet, they do not contribute to the cohomology of the BRST operator. The cohomology of the $W_{2,4}$ BRST operator is thus equivalent to that of the BRST operator of the bosonic string

$$
Q = \oint c\Big(T_X - b_2\partial c_2 - b_3\partial c_3 - b\partial c\Big) \ , \tag{13}
$$

where the central charge for T_X is $c_X = 30$. Hence the new realisation provides an embedding of the Virasoro string with $c = 30 - 4$ in the non-critical $W_{2,4}$ string.

We can also construct the critical $W_{2,4}$ string using this new realisation (8). When $t^2 = \frac{6}{5}$, the central charge of the realisation (8) takes the critical value $c = 172$. In this case, $c_X = 33$ and the central charge of the (b_1, c_1) system is -7. The critical realisation for the $W_{2,4}$ algebra can be straightforwardly obtained by performing a basis change of the linear $W_{1,2,4}$ algebra [9].

The critical BRST operator for $W_{2,4}$ can also be written in a graded form $Q = Q_0 + Q_1$ [5]. In terms of this realisation, the Q_0 operator is given by Eqn. (10) with $T_{\rm L}$ omitted; the Q_1 operator is given by

$$Q_1 = \oint \gamma\Big(r - \tfrac{1}{4} r^4 s^4 + \tfrac{1}{6} r^3 s^3\, b_1 c_1 + \hbox{quantum corrections}\Big)\ . \tag{14}$$

As in the case of the critical W_3 string discussed in Ref. [2], we expect that this operator can also be converted into a single term γr. Thus the cohomology of the critical $W_{2,4}$ BRST operator is equivalent to that of the bosonic string with the energy-momentum tensor $T_X + T_{c_1,b_1}$, where the central charges for T_X and T_{c_1,b_1} are 33 and -7 respectively.

Now let us consider $W_{2,6}$ strings. As we have shown in this paper, with the inclusion of a spin–1 current the $W_{2,6}$ algebra can be linearised only for central charge $c = 390$. The linearised $W_{1,2,6}$ algebra is given by Eqn. (1) with $s = 6$, $c_1 = 11$ and $h = -1$. A realisation can be easily obtained, given by

$$T_0 = T_X + 6r\partial s + 5\partial r\, s\ , \qquad W_0 = r\ , \qquad J_0 = rs\ . \tag{15}$$

The central charge for T_X is 28. The realisation for $W_{2,6}$ can then be obtained by substituting Eqn. (15) into Eqn. (7). Since the critical central charge for $W_{2,6}$ is 388, we need to use a non-critical $W_{2,6}$ BRST operator, with the Liouville sector contributing a central charge $c = -2$. The $W_{2,6}$ algebra becomes degenerate at central charge $c = -2$ [13], in the sense that the OPE of the spin–6 current with itself gives rise only to descendants of the spin–6 currents. Thus it is possible to set the spin–6 current to zero in the non-critical BRST operator for $W_{2,6}^{\rm M} \otimes W_{2,6}^{\rm L}$ at this central charge, leading to a $c_{\rm M} = 390$ non-critical $W_{2,6}$ BRST operator with a purely Virasoro Liouville sector, which is given by [13]

$$\begin{aligned}
Q_0 &= \oint c\Big(T + T_{\rm L} - 6\beta\partial\gamma - 5\partial\beta\,\gamma - b\partial c\Big)\ ,\\
Q_1 &= \oint \gamma\Big(2448\sqrt{\tfrac{41149461318}{13}}\, W_{\rm M} + 4282\, T_{\rm M}^3 + \tfrac{1390837}{13}\,\partial^2 T_{\rm M}\, T_{\rm M}\\
&\quad + \tfrac{1038100}{13}\,\partial T_{\rm M}\,\partial T_{\rm M} + \tfrac{6815257}{39}\,\partial^4 T_{\rm M} - \tfrac{1032462}{13}\, T_{\rm M}^2\,\beta\partial\gamma\\
&\quad + \tfrac{4301925}{13}\,\partial^2 T_{\rm M}\,\beta\partial\gamma + \tfrac{16634110}{13}\,\partial T_{\rm M}\,\partial\beta\,\partial\gamma + \tfrac{6653644}{13}\, T_{\rm M}\,\partial^2\beta\,\partial\gamma\,\gamma\\
&\quad - \tfrac{9980466}{13}\, T_{\rm M}\,\beta\partial^3\gamma\,\gamma - 1433975\,\partial^4\beta\,\partial\gamma + 2581155\,\partial^2\beta\partial^3\gamma\\
&\quad - 1433975\,\beta\partial^5\gamma\,\gamma - 1720770\,\partial\beta\,\beta\partial^2\gamma\,\partial\gamma\Big)\ ,
\end{aligned} \tag{16}$$

where the Liouville current $T_{\rm L}$ generates the Virasoro algebra at $c = -2$, and hence it can be realised as $T_{\rm L} = -b_1\partial c_1$. Substituting the new realisation (7) for $W_{2,6}$, with T_0, W_0 and J_0 given

by (15), we expect that the Q_1 operator can be transformed into a single term γr by a local canonical field redefinition. (Note that the classical terms in the Q_1 operators are $\gamma(r - \frac{1}{6}r^6 s^6)$ modulo an overall constant factor.) Thus the cohomology of this BRST operator is equivalent to that of the Virasoro string with energy-momentum tensor $T = T_X - b_1 \partial c_1$.

To summarise, we have shown in this paper that the bosonic string can be embedded into $W_{2,s}$ strings for $s = 4, 6$, extending previous results for $s = 1, 2$ and 3. The key feature that makes the embedding possible is that the realisation of the higher-spin current involves a term linear in a ghost-like field. The existence of such a linear term was implied by the fact the $W_{2,s}$ ($s = 3, 4, 6$) algebras can be linearised with the inclusion of a spin–1 current. Such a linearisation is possible for $W_{2,3}$ and $W_{2,4}$ at all values of central charge [1, 9]. In this paper, we showed that for the case of $W_{2,6}$, the linearisation is possible only when the central charge is 390. We found realisations in terms of ghost-like fields for the $W_{2,4}$ and $W_{2,6}$ algebras, and used these new realisation to construct the corresponding W strings. We argued that the associated BRST operators are equivalent to that of the Virasoro string. It would be interesting to extend these results to other W strings. The linearisation of the W_N algebra has been obtained recently in Ref. [14], which may provide new realisations for the embedding of the bosonic string. It would also be of great interest to investigate the nested embedding of the W_N string in the W_{N+1} string.

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